JOURNAL OF AIRCRAFT
Vol. 42, No. 4, July–August 2005

Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes should not exceed 2500 words (where a figure or table counts as 200 words). Following informal review by the Editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Some Characteristics of Supersonic Transport Trajectories

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Introduction

In a previous series of papers,[1–3] a trajectory optimization algorithm was developed and incorporated it into the NASA Ames Research Center ACSYNT aircraft design synthesis code.[4] The algorithm determines near-optimal fixed-range trajectories of supersonic transports. The method is based on singular perturbation theory. A parameter is inserted in the equations of motion in such a way that the equations are decoupled into three timescales. Each of the three sets of equations is solved by expanding all state and control variables to first order in the parameter, that is, by retaining the first two terms in the asymptotic expansions. The solutions are then matched to give uniformly valid approximations to the original equations of motion. The details of the approach are given in Refs. 5 and 6.

The slowest timescale consists of the weight/range dynamics. For the case of minimum fuel consumption, the zero-order solution of this system is the familiar Brequet cruise climb. The intermediate timescale consists of the energy dynamics; the zero-order solution in this case is the familiar energy climb path. For both timescales, the first-order solutions provide small corrections to the zero-order solutions. The fastest timescale consists of the altitude/flight-path angle dynamics; this timescale is not considered here.

A similar analysis was carried out in Ref. 7. The differences are that here we carry out the solutions to first order for both timescales, and we apply our algorithm to supersonic transports.

Transport trajectories are ideally suited for timescale separation because they naturally consist of climb, cruise, and descent segments. In Refs. 5 and 6, it was shown that first-order solutions give highly accurate results, even for high-performance aircraft.

The purpose of the present Note is to illustrate the trajectory algorithm by using it to study the properties of the trajectories of a contemporary supersonic transport design. The transport's parameters are listed in Table 1. Figure 1 shows its geometry, and Fig. 2 gives its lift-to-drag characteristics. The design is the same as that in Refs. 1–3.

The first issue investigated is that of the relationship between cruise Mach number M and range R. It was found in Refs. 1–3 that there are three local optimal cruise Mach numbers, at M 2.4, M 1.6, and M 0.95. The global optimum is M 2.4, the maximum value permissible for this design; however, it may be that at short

Presented as Paper 2000-4247 at the AIAA Guidance, Navigation, and Control Conference and Exhibit, Denver, CO, 14–17 August 2000; received 6 July 2004; revision received 31 August 2004; accepted for publication 2 September 2004.

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Table 1 Supersonic transport parameters

Parameter	Value	Parameter	Value
Gross weight	759,300 lb	Wing planform area	5500 ft^2
Fuel weight	416,685 lb	Wingspan	137.35 ft
Payload		Leading-edge sweep	48 deg
First-class passengers	30	Aspect ratio	3.43
Coach-class passengers	274	Body length	314 ft
Flight crew	2	Maximum M	2.4
Flight attendants	9	Maximum q	1000 lb/ft^2

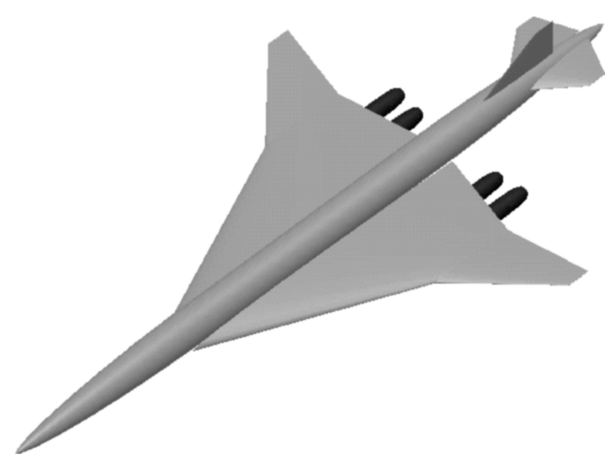

Fig. 1 Supersonic transport geometry.

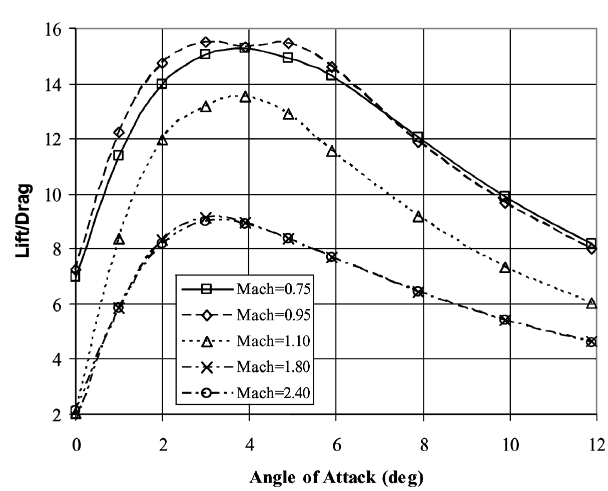

Fig. 2 Supersonic transport lift-to-drag ratio.

ranges one of the lower cruise speeds is better. The second issue is the effect on performance caused by the need to restrict flight to subsonic speeds during flight over land, where sonic booms are prohibited. Thus, we consider mixed supersonic/subsonic flights. Finally, we determine the payload–range curve of the aircraft.

References 1–3 considered minimum time; minimum fuel; and, by using a weighted sum of these criterion; minimum direct operating cost trajectories. It was found that minimum cost trajectories were very similar to those of minimum fuel (minimum time trajectories being quite different), and, consequently, we will consider only the minimum fuel case in this Note.

The fixed-range trajectory problem is bounded by several state and control constraints. First, the M upper limit is 2.4, and the dynamic pressure q upper limit is 1000 lb/ft^2. The altitude h lower limit is 1500 ft, whereas the h upper limit is 60,000 ft. Also, throttle setting π is bounded by idle and maximum thrust, $\pi = 1$.

Optimal Cruise Mach Number for Minimum Fuel

We first compare trajectories cruising at the three locally optimum Mach numbers to see which one consumes less fuel. The comparison is made for ranges of 500, 1000, and 2000 n miles. In practice, the amount of fuel loaded on to the aircraft depends on the range, but for comparison purposes it is sufficient to hold the initial weight fixed for all cases.

Figure 3 shows the 500-n mile range trajectories. The only trajectory to reach cruise for this range is the Mach 0.95; the Mach 1.6 and 2.4 cases consist solely of climb and descent. The cruise Mach 1.6 and 2.4 trajectories both exhibit substantial dives in their climb paths. These dives occur at $M1$ and are due to the transonic drag rise. In effect, the aircraft dives to get through this region of high drag as quickly as possible. In Ref. 1, it is shown that these dives may be accomplished approximately by maneuvers acceptable to passengers. The jumps at the top of these trajectories are due to the nonsmoothness of the aerodynamic and propulsive data that were used.

The 1000-n mile trajectories are shown in Fig. 4. Here the only trajectory that does not possess a cruise segment is the Mach 2.4 case.

Finally, Fig. 5 shows the 2000-n mile trajectories. In this case, all three trajectories reach cruise.

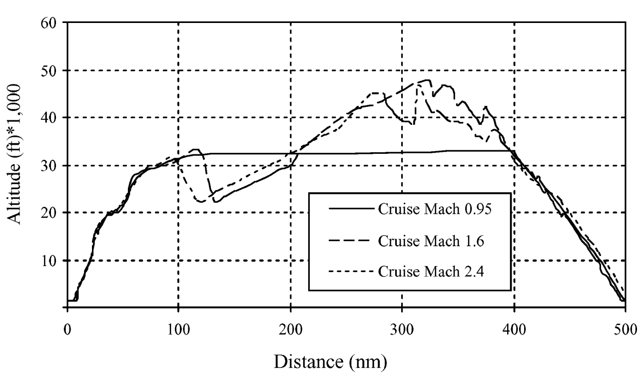

Fig. 3 Altitude vs distance, 500-n mile range.

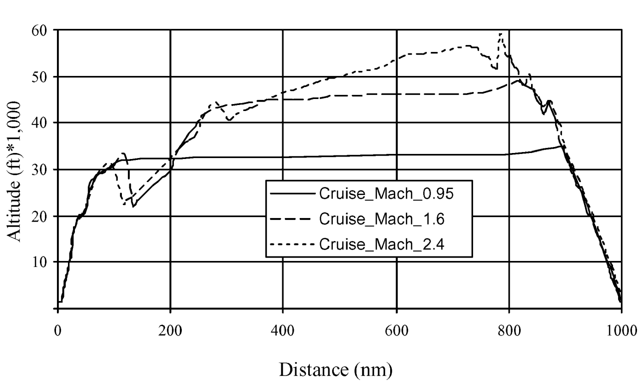

Fig. 4 Altitude vs distance, 1000 n mile range.

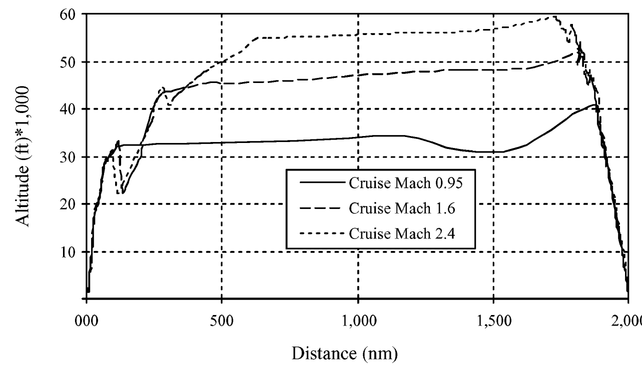

Fig. 5 Altitude vs distance, 2000 n mile range.

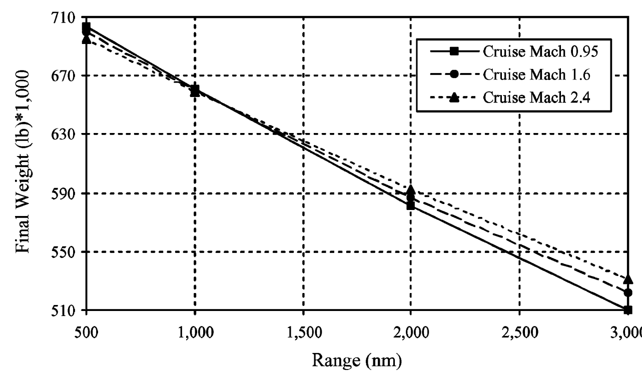

Fig. 6 Aircraft final weight as function of range.

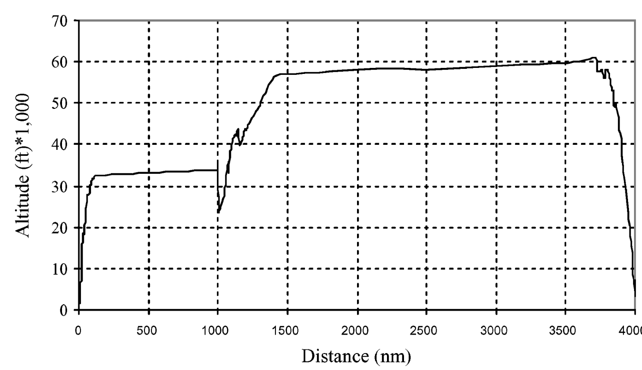

Fig. 7 Altitude vs Distance.

Figure 6 shows final vehicle weight as a function of range for the three cruise Mach numbers. Note that all three curves cross at $R = 1000$ n mile. Trajectories with 0.95-Mach cruise are optimal for ranges less than 1000 n mile, and trajectories with 2.4-Mach cruise are optimal for ranges greater than 1000 n mile. Trajectories with 1.6-Mach cruise are optimal for $R = 1000$ n mile, but so are the other trajectories. The optimality of cruise at M 2.4 is not surprising because M 2.4 is the design point of the aircraft. These results are due to an obvious tradeoff. Attaining the optimal M 2.4 cruise point requires a lengthy climb (a relatively inefficient flight segment). Thus, the longer climb pays off only if a considerable fraction of the total time is spent at cruise.

Mixed Supersonic and Subsonic Flights

Because of the sonic boom they create, commercial supersonic vehicles are banned from traveling over land at supersonic speed. Therefore, a flight over land and sea will consist of mixed cruising at supersonic and subsonic speeds. For our aircraft model and for long distance flight, supersonic cruise is at Mach 2.4 and subsonic cruise is at Mach 0.95. Figure 7 shows a 4000-n mile mixed-speed trajectory. The aircraft first cruises at Mach 0.95 for 1000 n mile, and then cruises at Mach 2.4 for the remaining 3000 n mile. An

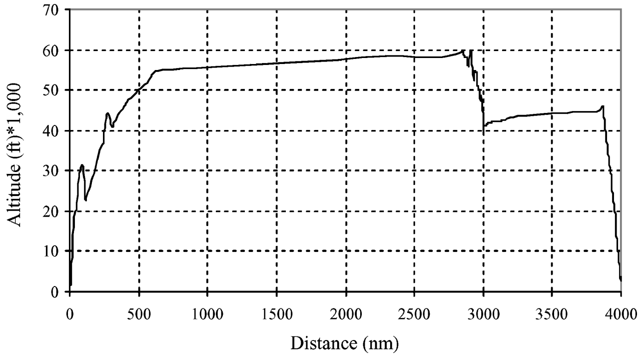

Fig. 8 Altitude vs Distance.

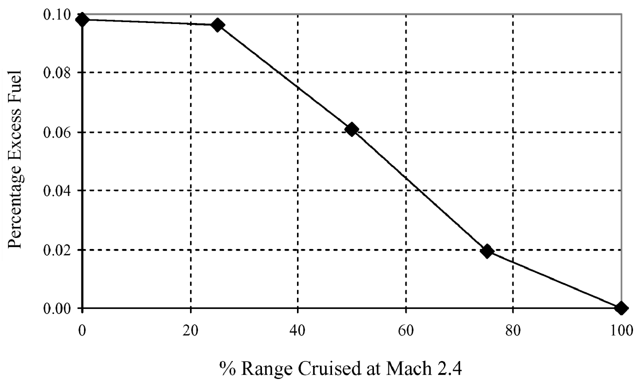

Fig. 9 Normalized extra fuel consumed vs percent range cruised at Mach 2.4.

energy layer is incorporated between the two cruises to transition the vehicle from Mach 0.95 to 2.4. Note that there is a transonic dive in this transition. Figure 8 shows a similar trajectory, one that begins cruising at Mach 2.4 and ends cruising 1000 n mile at Mach 0.95.

It was found that the final weight of the vehicle and the total flight time were almost exactly the same for the two trajectories. Thus, performing the subsonic cruise before or after the supersonic cruise does not affect weight and time performance.

Figure 9 shows that, as the percentage of the range cruised supersonically decreases (and the percentage cruised subsonically increases accordingly), the fuel consumed goes up substantially. The difference in weight between all subsonic and all supersonic cruise amounts to over 120 passengers at a range of 4000 n mile. Of course, the flight time also rises sharply as the percentage of supersonic cruise decreases.

Payload–Range Curve

The payload–range curve in terms of number of passengers is shown in Fig. 10. The average weight of each passenger with baggage is assumed to be 210 lb. The payload range curve contains three segments. The section of the curve to the left of point 1 represents a vehicle that begins its flight with a full payload of passengers and fuel tanks partially full, the fuel load increasing as the range increases. The section of the curve between points 1 and 2 represents a vehicle that begins its flight with a partial load of passengers and a partial

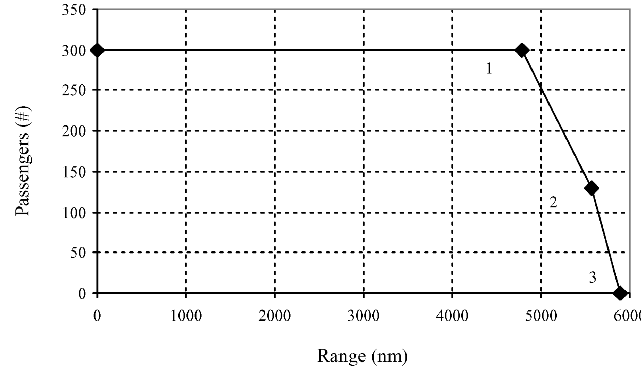

Fig. 10 Payload (number of passengers) vs range curve.

load of fuel, such that the gross takeoff weight is at its maximum. Here, to achieve greater ranges, passengers must be swapped out in favor of adding fuel. Finally, the section of the curve between points 2 and 3 represents a vehicle with a full tank of fuel and a partial load of passengers. Now, increased range can only be achieved by taking passengers off of the air plane to decrease its weight. Figure 10 shows that the maximum range is about 5800 n mile, and the range with all seats filled is about 4800 n mile.

Conclusions

A previously developed trajectory algorithm has been applied to a contemporary supersonic transport design to investigate the nature of its optimal trajectories. It was found that the optimal cruise Mach number for flights of less than 1000 n mile (not of much interest for supersonic transports) is M 0.95. At longer ranges, the optimal cruise Mach number is M 2.4, the maximum allowable.

Mixed supersonic/subsonic flights were investigated. Results showed that performance is the same (in terms of flight time and fuel consumption) if the subsonic flight segment is at the beginning or at the end of the flight. The fuel consumption as well as the flight time increases significantly as the percentage of subsonic flight increases.

Finally, the payload–range curve of the aircraft was determined. This showed that the maximum range is 5800 n mile and that the range with a full passenger load is 4800 n mile.

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